

WHAT IS THE "RIGHT" METRIC FOR THE MODELLING OF FEATURE MAPS IN THE VISUAL CORTEX?

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1. INTRODUCTION

Spike rates of neurons in the visual cortex depend on the type of visual stimulation. The specific properties of each neuron are typically investigated by exposing the subject to a stimulus with a certain set of features, i.e. orientation, position in the visual field and others. The activity of this neuron is then recorded with respect to the presented stimulus. Optical imaging experiments show not only that single neurons show a preference to certain features but also that each point of the cortex surface is typically activated by a set of specific preferred stimulus features. Thus, neurons with similar stimulus preferences are grouped together at one small area on the cortical surface. These preferences change typically continuously as a function over the position of the cortex surface and form a structure which shows a 2 dimensional pattern ("map") of a certain wavelength. The complete cortical area roughly covers the whole feature space, i.e. all possible combinations of features[7].

These patterns are the biggest cortical structures that might be affected by early experiences and thus, they play a major role in the nurture versus nature discussion. The patterns partly resemble structures that appear in physical pattern formation processes, and have been assumed to be a result of a similar process. Hence, the coverage of the stimulus space is optimized in a learning process that depends on the stimulus statistics. Therefore, it seems to be plausible to assume that the visual cortex undergoes an optimization process of two competing optimization criteria, that are (1) the "optimal folding" of the map into the complete stimulus space and (2) a "neighbourhood preservation". Optimal folding means that there is for every point in the stimulus space, a corresponding point on the map that is as close as possible (in the sense of distance measure). Neighbourhood preservation means that the developmental process results in neighboring neurons that prefer similar stimuli, i.e. show a set of preferred features that is similar. (For review for models of the visual cortex please consult [6].)

Both criteria, optimal coverage and neighbourhood preservation are dependent on assumptions of the underlying metrics of the stimulus space, its features respectively. In previous works consequences of modifications of these assumptions[1, 2] were investigated. It could be shown that the exact design of the metrics had a crucial impact on the patterns that finally emerged as most stable after a prolonged formation process.

There are indications that such a prolonged developmental process actually takes place. One indication results from the analysis of singular points on the map of orientation preference [8]. The orientation maps from certain species show a lower density of singular points ("pinwheels") than one would expect if the maps would remain in an initial state that emerged spontaneously from a dynamic instability.

In the following section we present a model that is intended to resemble the cortical areas 17, 18 alias V1 and V2. Presently it is reduced to two stimulus features that is the position of the stimulus in the visual field and its orientation. We briefly outline this approach and report the results.

In this initial approach the metric explicitly relates to the stimulus set. This seems improvable for several reasons. Neurons experience only activity statistics, that are spike rates and the spike timings of connected neurons. Thus, the spike-encoded information from afferent pathways and laterally connected neurons is the only criterium for the local optimization process. In this way a statistical model of the stimulus set and a metric that is derived from these statistical measures may be useful to provide a means to design new models of the process, and – more interestingly – to analyze if indeed maps that have been found in animals show signatures of an optimization process of any kind.

Some preliminary considerations of this point are outlined in the discussion section of this work. We also discuss in which way the symmetries derived for the previous approach might apply to the new type of approach.

2. A FEATURE METRIC BASED ON SYMMETRY CONSIDERATIONS

The following considerations describe the derivation of a metric between points in the feature space. Our intention is two fold: On one hand the considerations are the bases of a model that results in a non-trivial example of how the formation of feature maps can actually be affected by a metric, and its variation. On the other hand the same considerations may be useful to derive a model that uses stimuli distributions, and in this way simplify the calculation of a metric of those.

In the following we assume a four dimensional feature space: $\{x, y, z_1 = |z| \sin(2\phi), z_2 = |z| \cos(2\phi)\}$, where x and y indicate the the position of the stimulus in the visual field, ϕ the orientation of the stimulus and $|z|$ the orientation strength. Since orientation is a π -periodic feature the argument to the sine is multiplied by a factor of two. The used encoding of the orientation strength and orientation yields a valid mapping that is topologically correct, i.e. for zero orientation strength $|z| = 0$ all preferred orientations coincide to one coordinate ($z_1 = z_2 = 0$), all orientations are in the neighborhood of zero orientation strength.

A Riemannian metric should yield a 4×4 metric tensor g_{ij} . In the following symmetric operations are outlined for which we assume the metric should be invariant.

- The tensor should be invariant under translations of x and y . This assumption is almost true in the part of the visual cortex that corresponds to foveal vision. Further, the change of the size of the projection is much slower than the typical wavelength of the pattern of the preferred orientation. Thus, for a simple approach we are going to neglect it.
- The metrics should be invariant against the inversion of the position difference (dx, dy) between two stimuli.
- The metrics should be invariant under the combined rotation of position x, y and orientation preference z_1, z_2 .

It should be noted that the number of parameter functions can be further reduced by regauging (redefining) the orientation strength.

In the following we use Riemannian metrics that all obey the above mentioned symmetry constraints; the metric tensors are the following:

$$\begin{pmatrix} 1 + \alpha z_1 & \alpha z_2 & 0 & 0 \\ \alpha z_2 & 1 - \alpha z_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The parameter α modulates a coupling strength between the feature of orientation preference and retinal position. Experimental results indicate [10] that the two features are indeed coupled in maps of the visual cortex. In addition, in most high-dimensional models (e.g. [9]) this kind of coupling appears implicitly. The same kind of coupling has also been considered in [4].

The metric has been built into a modified 2 dimensional elastic net approach where the energy function

$$E = -A \left\langle \log \left(\sum_{\mathbf{i} \in \mathcal{N}} \exp \left(-\frac{d^2(\mathbf{v}, \mathbf{w}_{\mathbf{i}})}{2\sigma^2} \right) \right) \right\rangle_{\mathbf{v} \in \mathcal{V}} + B \sum_{\mathbf{i} \in \mathcal{N}} \sum_{|\mathbf{i}-\mathbf{j}|=1} d^2(\mathbf{w}_{\mathbf{i}}, \mathbf{w}_{\mathbf{j}})$$

was optimized by gradient descent. \mathcal{N} was a periodic two dimensional grid and \mathcal{V} the stimulus space. The parameters A and B control the stiffness of the map and were chosen to keep the dynamics in state that showed only weak orientation preferences [2].

In this model each node of the net corresponds to a point of the cortex. The topology of the cortex is reflected by the topology of the network. For the simulations only the square components of the Taylor series of the metrics were used. The parameter α was kept constant during each simulation; several values of α were tested. The results are depicted in Fig. 1. For a metric with no coupling ($\alpha = 0$) we get a map of rolls that is completely free of singular points, whereas for a map with this coupling shows a regular pattern containing between 3 and 4 singularities per wavelength.

Similar investigations have been done with a modified Swift-Hohenberg model[11]. The results from the modified Swift-Hohenberg model show similar patterns as in the elastic net approach.

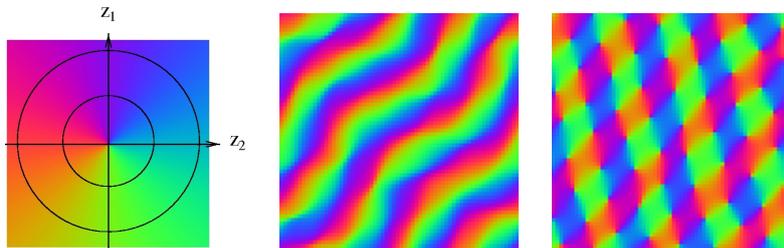


FIGURE 1. Left: Color code for the orientation preference. Middle: Map evolving after a long learning period from a metric with $\alpha = 0$. Right: A metric evolving from a metric with $\alpha = 0.3$. Both approaches were done with a modified elastic net approach [2].

3. DISCUSSION AND OUTLOOK: CROSS MODAL MAPS

The aim of our methods is to find the general principle of the formation process in various cortical areas, whose nature is unclear so far. The above outlined work

briefly summarized work that investigated a simplified set of exemplary metrics. Further the work reported numerical results from which we see that the structure of a feature map depends on the metric that describes a distance measure between two sets of features. Cortical neurons obviously use some local, statistical method for the individual learning process, thus a similar method seems appropriate for the adaptation process of the each node of the model.

A different field of interest for maps of feature spaces are cross modal maps. Cross modal maps are used for example in robotics to organize combined representations of several sensory inputs. They can be used for models of proprioceptive learning. The introduction of an arbitrary mixed feature space of two senses can be replaced by a more general unsupervised learning process. For this purpose a statistic approach seems to be appropriate.

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